1/31/18 "Failing to prepare, is preparing to fail."-Anonymous

HW: "Shifting Functions" Homework section

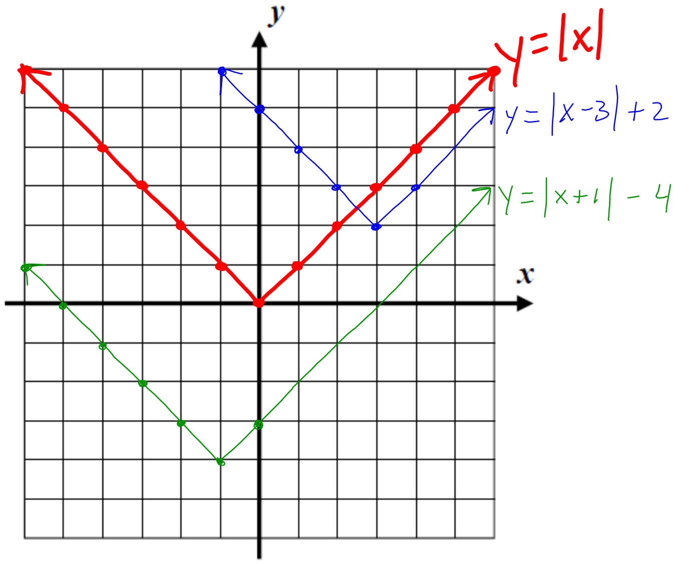
AIM: How do we identify shifts of functions?

Warm Up:

Exercise #1 part (a) on the handout

***Exercise* #1:** Consider the functions $y = |x|$, $y = |x-3| + 2$, and $y = |x+1| - 4$.

(a) Without the use of your calculator, graph $y = |x|$ on the axes provided. Label its equation.



(b) Using your calculator to generate a table of values, graph the other two absolute value functions above and label each with its equation.

$y = |x-3| + 2$

x	y
-1	6
0	5
1	4
2	3
3	2
4	3

$y = |x+1| - 4$

x	y
-6	1
-5	0
-4	-1
-3	-2
-2	-3
-1	-4
0	-3

(c) How would the graph of $y = |x|$ be shifted in order to produce the graph of $y = |x-6| - 8$?

right 6 down 8

✱ Inside: effects the x-values (opposite of what is shown) (left / right)
$+$ $-$

✱ Outside: effects y-value (do what is shown) (up / down)
$+$ $-$

up/down

VERTICAL AND HORIZONTAL SHIFTING

1. **Vertical Shifting**: The function $f(x)+k$ shifts the function up by $|k|$ units for $k>0$ and down $|k|$ units for $k<0$.
   up
   $f(x)-k$ down

left/right

2. **Horizontal Shifting**: The function $f(x+k)$ shifts the function left $|k|$ units for $k>0$ and right $|k|$ units for $k<0$.
   left $f(x-k)$ right

***Exercise* #2:** The function $f(x)$ is shown on the grid below. A second function, g, is defined by $g(x)=f(x-3)+1$.

right 3 units

up 1 unit

(a) What is the value of $g(0)$? Show how you arrived at your answer.

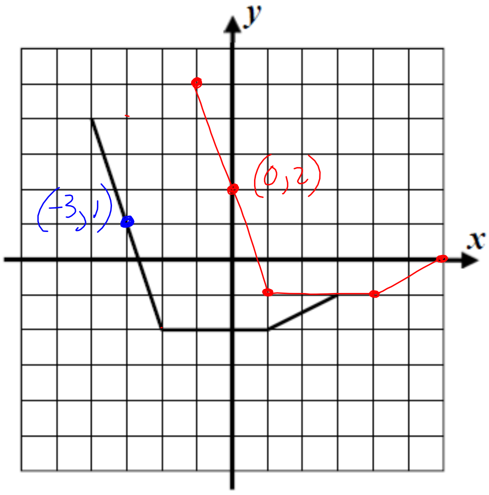
$g(x)=f(x-3)+1$

$g(0)=f(0-3)+1$

$=f(-3)+1$

$=(1)+1$

$g(0)=2$



(b) Identify how the graph of f has been transformed to produce the graph of g and sketch it on the grid.

Right 3 and Up 1

We can use these shifting patterns in a variety of ways because they apply to all types of functions.

***Exercise* #3:** A function, $f(x)$, has a domain of $-3 \le x \le 10$ and a range of $y \le 22$. What are the domain and range of the function $f(x+7)+10$? Explain how you arrived at your answers.

left 7 (subtract 7 from x)

up 10 (add 10 to y)

$f(x+7)+10$

Domain: $-10 \le x \le 3$

Range: $y \le 32$

original $-3 \le x \le 10$
$-7 \quad -7$
$-10 \le x \le 3$

$y \le 22$
$+10$
$y \le 32$

Recognizing shifts of other, simpler functions can help us identify prominent characteristics and compare them. The location of turning points is especially helpful.

***Exercise* #4:** Given the quadratic function $f(x)=(x-4)^2-5$ answer the following questions.

(a) How has the simple quadratic $y=x^2$ been shifted to produce the graph of $f(x)$?

Right 4 units

Down 5 units

(b) Given that $y=x^2$ has a turning point at the origin, $(0,0)$, where must the turning point of f lie?

vertex

$(0,0)$

$+4 \quad -5$

$\boxed{(4,-5)}$

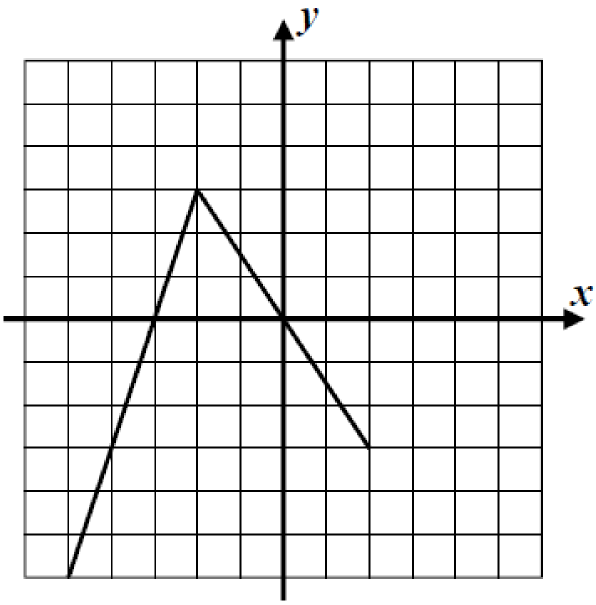
(c) Sketch f below and give the domain interval over which f is increasing.

(d) Which has a lower minimum value, the function f or the function $g(x)=|x-6|-10$? Explain your choice.

One of the hardest things for students to grasp is the horizontal shift, which appears to work opposite of what we would expect. Let's take a look at a shift that is purely horizontal.

***Exercise* #5:** The graph of $f(x)$ is shown below. The function $g(x)$ is defined by $g(x) = f(x-2)$.

(a) Show that $x = -1$ and $x = 2$ are zeroes of the function g.



(b) Evaluate each of the following using the definition of g and then create a plot of g on the same set axes.

$g(-3) =$ $g(0) =$ $g(4) =$